

Q1 Basic Continuous Probability

2 Points

Let $F(x)$ be the cumulative distribution function of a random variable X , and $f(x)$ be its probability density function.

Q1.1 CDF

1 Point

True or False: $F(x) = \frac{df(x)}{dx}$ for any $x \in \mathbb{R}$

☐ True

☒ False

Explanation

$F(x) = \mathbb{P}[X \leq x] = \int_{-\infty}^x f(z)dz$, so differentiating both sides with respect to x gives us $f(x) = \frac{dF(x)}{dx}$. Note that F and f have been flipped in the question.

Q1.2 Joint distribution

1 Point

True or False: $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)dx dy = 1$.

☒ True

☐ False

Explanation

Analogous to probability summing to 1 with discrete random variables.

Q2 Continuous Distributions - Uniform

2 Points

Let X be a uniform r.v. on the interval $[0, p]$. Which of the following are true statements?

Q2.1

1 Point

$$f_X(x) = \frac{1}{p}, \forall x \in \mathbb{R}$$

☐ True

☒ False

Explanation

The PDF is incorrect, since it has to be 0 outside of the range $[0, p]$.

Q2.2

1 Point

$$\mathbb{E}[X] = \frac{p}{2}$$

☒ True

☐ False

Explanation

$$\mathbb{E}[X] = \int_0^p \frac{1}{p} x dx = \frac{1}{p} \frac{p^2}{2} = \frac{p}{2}$$

Q3 Continuous Distribution - Exponential

4 Points

Let X be a exponential r.v. with parameter λ . Which of the following are true statements?

Q3.1

1 Point

$f_X(x) = \lambda e^{-\lambda x}$ for $x \geq 0$, and 0 elsewhere

☒ True

☐ False

Explanation

By definition.

Q3.2

1 Point

For $t \geq 0$, $\mathbb{P}[X > t] = e^{-\lambda t}$

☒ True

☐ False

Explanation

$$\mathbb{P}[X > t] = \int_t^{\infty} \lambda e^{-\lambda x} dx = (-e^{-\lambda x}) \Big|_{x=t}^{\infty} = e^{-\lambda t}$$

Q3.3**1 Point**

For $t \geq 0$, $\mathbb{P}[X \geq t] > \mathbb{P}[X > t]$

☐ True

☒ False

Explanation

The two sides differ by the quantity $\mathbb{P}[X = t]$, which is equal to zero for continuous random variables. So the two sides are actually equal.

Q3.4**1 Point**

$\mathbb{E}[X] = \lambda$

☐ True

☒ False

Explanation

The expected value of an exponential random variable is $\frac{1}{\lambda}$.

Q4 Integration Review

3 Points

As we stated in HW 1, we expect you to compute derivatives, integrals, and double integrals. Please take the time to compute these integrals by hand.

Q4.1

1 Point

$$\int_0^{\infty} x e^{-x} dx$$

$$=1+-0$$

Explanation

We want to find the integral of a product of functions, which Integration by Parts is appropriate for.

Let $u = x$, $dv = e^{-x} dx$. Then $du = dx$, $v = -e^{-x}$.

Integration by Parts gives us $\int u dv = uv - \int v du$.

$$\begin{aligned}\int x(e^{-x} dx) &= x(-e^{-x}) - \int -e^{-x}(dx) \\ &= -xe^{-x} + \int e^{-x} dx \\ &= -xe^{-x} - e^{-x}\end{aligned}$$

Integrating from 0 to ∞ :

$$\begin{aligned}\int_0^{\infty} x e^{-x} dx &= \lim_{b \rightarrow \infty} \int_0^b x e^{-x} dx \\ &= \lim_{b \rightarrow \infty} [-xe^{-x} - e^{-x}]_{x=0}^{x=b} \\ &= \lim_{b \rightarrow \infty} [(-be^{-b} - e^{-b}) - (-0e^{-0} - e^{-0})] \\ &= 0 + 1 \\ &= 1\end{aligned}$$

Note: Notice that the integral $\int_0^{\infty} x e^{-x} dx$ is precisely the expectation of an $\text{Exp}(1)$ random variable.

Q4.2**1 Point**

$$\frac{1}{b^5} \int_0^b 9(bx)^2 dx, \text{ for all constants } b > 0$$

$$=3+-0$$

Explanation

First, notice that the variable of integration is x . The other symbol, b , behaves like a **constant**.

$$\begin{aligned} \frac{1}{b^5} \int_0^b 9(bx)^2 dx &= \frac{1}{b^5} 9b^2 \int_0^b x^2 dx \\ &= \frac{9}{b^3} \left[\frac{1}{3} x^3 \right]_{x=0}^{x=b} \\ &= \frac{9}{b^3} \left[\frac{1}{3} b^3 - \frac{1}{3} 0^3 \right] \\ &= 3 \end{aligned}$$

Note: Treating symbols that aren't the variable of integration like **constants** is relevant for computing double integrals!

Q4.3**1 Point**

$$\int_0^4 \int_0^3 (x^2 + xy) dy dx$$

If you're stuck, evaluate the inner integral first, keeping x as a **constant**.

$$=100+-0$$

Explanation

First, evaluate the inner integral, treating x as a **constant**.

$$\begin{aligned}\int_0^4 \int_0^3 (x^2 + xy) dy dx &= \int_0^4 [x^2 y + \frac{1}{2} xy^2]_{y=0}^{y=3} dx \\ &= \int_0^4 [(x^2(3) + \frac{1}{2}x(3)^2) - (x^2(0) + \frac{1}{2}x(0)^2)] dx \\ &= \int_0^4 (3x^2 + \frac{9}{2}x) dx\end{aligned}$$

Now let's evaluate the outer integral.

$$\begin{aligned}\int_0^4 (3x^2 + \frac{9}{2}x) dx &= [x^3 + \frac{9}{4}x^2]_{x=0}^{x=4} \\ &= [(4^3 + \frac{9}{4}4^2) - (0^3 + \frac{9}{4}0^2)] \\ &= 64 + 36 \\ &= 100\end{aligned}$$