Q1 Conditional and Total Expectation

2 Points

Say the average rainfall experienced by a person from Berkeley is 8 inches a year, and the average rainfall in San Francisco is 12 inches per year. Suppose also that when you pick someone from the two cities, they come from Berkeley with probability .6 and from San Francisco with probability .4.

Q1.1

1 Point

What is the expected amount of rainfall experienced by the person if they are from Berkeley?

- ( ) 0
- (•) 8
- ( ) 9.6
- ( ) 12

Explanation

This was given in the question - the average rainfall experienced by a person from Berkeley is 8 inches a year.

Q1.2
1 Point

What is the expected amount of rainfall experienced by the person?

- ( ) 0
- ( ) 8
- (•) 9.6
- ( ) 12

Explanation

Let B be the event that the person is from Berkeley and S be the event that the person is from San Francisco, and R be the random variable for amount of rainfall experienced. Recall that $\mathbb{E}[R \mid B] = 8$ from the previous part, and we also have $\mathbb{E}[R \mid S] = 12$. We also have $\mathbb{P}[B] = 0.6$, $\mathbb{P}[S] = 0.4$. To get $\mathbb{E}[R]$, use total expectation: $\mathbb{E}[R] = \mathbb{E}[R \mid B]\mathbb{P}[B] + \mathbb{E}[R \mid S]\mathbb{P}[S] = 8 \cdot 0.6 + 12 \cdot 0.4 = 9.6$.

Q2 Variance of 4-sided die

2 Points

Let X represent the result of 1 roll of a fair 4-sided die.

Q2.1

1 Point

What is $\mathbb{E}[X^2]$?

- (selected) $\frac{15}{2}$
- 3
- $\frac{9}{2}$
- 15

Explanation

This is equal to $\sum_{i=1}^{4} i^2 \mathbb{P}[X = i] = \frac{1}{4}(1^2 + 2^2 + 3^2 + 4^2) = \frac{15}{2}$.

Q2.2

1 Point

What is $\mathrm{Var}(X)$?

- 5
- $\frac{5}{2}$
- $\frac{15}{8}$
- (selected) $\frac{5}{4}$

Explanation

This is equal to $\mathbb{E}[X^2] - \mathbb{E}[X]^2 = \frac{15}{2} - (\frac{5}{2})^2 = \frac{30}{4} - \frac{25}{4} = \frac{5}{4}$.

Alternatively, you could derive that the variance of a discrete $\mathrm{Uniform}\{1, \ldots, n\}$ random variable is $\frac{n^2-1}{12}$ so the variance here is $\frac{4^2-1}{12} = \frac{15}{12} = \frac{5}{4}$.

Q3 Variance, and Covariance

3 Points

Let $Y = Y_1 + Y_2 + Y_3 + Y_4$ represent the number of heads from tossing 4 fair coins, where each Y_i is an indicator representing whether the ith toss was a head (1 for head, 0 for tail).

Q3.1

1 Point

What is $\mathrm{Var}(Y)$?

- (selected) 1
- $\frac{3}{2}$
- 2
- $\frac{1}{2}$

Explanation

Since all the Y_i are independent, $\mathrm{Var}(Y) = \mathrm{Var}(\sum_{i=1}^{4} Y_i) = \sum_{i=1}^{4} \mathrm{Var}(Y_i) = \sum_{i=1}^{4} \mathrm{Var}(Y_i) = 4 \cdot \mathrm{Var}(Y_1) = 4 \cdot (\frac{1}{4}) = 1.$

Important note: "linearity" of variance does not hold in general! This is a special case since all the Y_i are independent, so their *covariances* are all 0. Usually, you will have to use bilinearity of covariance to "FOIL" out sums of random variables, and calculate the individual covariances between terms.

Q3.2
1 Point

What is $\mathrm{Cov}(Y_1, Y_2)$?

- (selected) 0
- $\frac{1}{2}$
- $\frac{1}{4}$
- $-\frac{1}{4}$

Explanation

Since all the Y_i are independent, $\mathrm{Cov}(Y_1, Y_2) = 0$. Mathematically, we have that $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$ for independent RVs, so $\mathrm{Cov}(Y_1, Y_2) = \mathbb{E}[Y_1 Y_2] - \mathbb{E}[Y_1]\mathbb{E}[Y_2] = 0$.

Q3.3
1 Point

What is $\mathrm{Cov}(Y, Y_1)$?

- 0
- $\frac{1}{2}$
- (selected) $\frac{1}{4}$
- $-\frac{1}{4}$

Explanation

By bilinearity of covariance, we have $\mathrm{Cov}(Y, Y_1) = \mathrm{Cov}(Y_1, Y_1) + \mathrm{Cov}(Y_2, Y_1) + \mathrm{Cov}(Y_3, Y_1) + \mathrm{Cov}(Y_4, Y_1) = \mathrm{Cov}(Y_1, Y_1) = \mathrm{Var}(Y_1) = \frac{1}{4}$ since Y_1 is independent from all other Y_i's.