

Q1 Random variables

2 Points

Consider a sample space Ω with $\mathbb{P}: \Omega \rightarrow \mathbb{R}$ and a random variable $X: \Omega \rightarrow \mathbb{R}$.

Q1.1

1 Point

What is $\mathbb{P}[X = x]$?

- ☐ $\sum_{\omega \in \Omega} \mathbb{P}[\omega]$
- ☒ $\sum_{\omega \in \Omega: X(\omega) = x} \mathbb{P}[\omega]$

Explanation

The first expression is 1. The second is summing over all sample points where $X(\omega) = x$, which is the probability that $X(\omega) = x$ over the possible choices of ω .

Q1.2

1 Point

If $x \neq y$, then $\mathbb{P}[X = x] + \mathbb{P}[X = y] = \sum_{\omega \in \Omega: X(\omega) \in \{x, y\}} \mathbb{P}[\omega]$.

- ☒ True
- ☐ False

Explanation

This is because $X(\omega)$ is a function, and no single ω can have two different values of $X(\omega)$.

Q2 Distribution

5 Points

Consider flipping a fair coin three times. Define the random variable X corresponding to the number of heads we flip.

Q2.1

1 Point

What is the number of sample points in the sample space?

- ☐ 0
- ☐ 1
- ☐ 2
- ☐ 4
- ☒ 8

Explanation

The number of sample points is $2 \times 2 \times 2 = 8$.

Q2.2

1 Point

What is $\mathbb{P}[X = 1]$?

- ☐ 0
- ☐ $1/7$
- ☐ $3/7$
- ☒ $3/8$
- ☐ $1/2$
- ☐ $5/7$
- ☐ 1

Explanation

$X = 1$ is the event that there is one heads. This has 3 possibilities out of 8 in total.

Q2.3**1 Point**

What is $\mathbb{P}[X = 2 \mid X \geq 1]$?

- ☐ 0
- ☐ 1/7
- ☒ 3/7
- ☐ 3/8
- ☐ 1/2
- ☐ 5/7
- ☐ 1

Explanation

There are 3 outcomes with two heads out of 7 outcomes with at least one heads.

Q2.4**1 Point**

What is $\mathbb{P}[X = 0 \mid X \geq 1]$?

- ☒ 0
- ☐ 1/7
- ☐ 3/7
- ☐ 3/8
- ☐ 1/2
- ☐ 5/7
- ☐ 1

Explanation

If there is at least one heads, then we cannot have $X = 0$.

Q2.5

1 Point

Does X follow a binomial distribution?

☒ Yes

☐ No

Explanation

We are repeating 3 trials with probability of success 0.5.

Q3 Binomial vs Bernoulli distribution

1 Point

Let n be a large integer and $p \in [0, 1]$ be a parameter. Assume X is a discrete random variable with probability density function $\mathbb{P}[X = i] = \binom{n}{i} p^i (1 - p)^{n-i}$ for each $i = 0, 1, \dots, n$.

Then the distribution of X is

☒ Binomial distribution

☐ Bernoulli distribution

Explanation

See note.

Q4 Joint distributions

1 Point

Let X, Y, Z be random variables on $\{0, 1\}$. Assume

- the marginal distribution of X is uniform over $\{0, 1\}$, and same for Y, Z ;
- the joint distribution of (X, Y) is uniform over $\{0, 1\}^2$, and same for $(X, Z), (Y, Z)$.

Then the joint distribution of (X, Y, Z) must be uniform over $\{0, 1\}^3$.

☐ True

☒ False

Explanation

Let X and Y be independent unbiased coins and define $Z = X + Y \pmod{2}$. Then you can verify that the conditions hold, but (X, Y, Z) can never be $(0, 0, 1)$.

Q5 Expectation

2 Points

Let X be the outcome of one six-sided die roll and Y be the outcome of a coin flip (i.e., $Y = 1$ corresponds to heads and $Y = 0$ corresponds to tails).

Q5.1

1 Point

What is $\mathbb{E}[X]$?

- ☐ 1
- ☐ 2.75
- ☒ 3.5
- ☐ 4
- ☐ 5.5
- ☐ 6

Explanation

The outcomes of a fair die roll have equal probabilities over 1, 2, 3, 4, 5, 6. Therefore the expectation is $(1 + 2 + 3 + 4 + 5 + 6)/6 = 3.5$.

Q5.2

1 Point

What is $\mathbb{E}[X + 4Y]$?

- ☐ 1
- ☐ 2.75
- ☐ 3.5
- ☐ 4
- ☒ 5.5
- ☐ 6

Explanation

$\mathbb{E}[Y] = 0.5$. Then $\mathbb{E}[X + 4Y] = \mathbb{E}[X] + 4\mathbb{E}[Y] = 3.5 + 4 \cdot 0.5 = 5.5$.