Q1 Sample Spaces

5 Points

For some experiment, we have a sample space $\Omega = \{1, 2, 3\}$. State whether each function is a valid probability function.

Q1.1

1 Point

$\mathbb{P}[1] = -0.2, \mathbb{P}[2] = 0.6, \mathbb{P}[3] = 0.6$

- ○ True
- ◉ False

Explanation

Probabilities must be nonnegative.

Q1.2

1 Point

$\mathbb{P}[1] = 0, \mathbb{P}[2] = 0, \mathbb{P}[3] = 1$

- ◉ True
- ○ False

Explanation

The probabilities are non-negative and sum to 1.

Q1.3
1 Point

$\mathbb{P}[1] = 1, \mathbb{P}[2] = 1, \mathbb{P}[3] = 0$

- ○ True
- ◉ False

Explanation
The probabilities do not sum to 1; therefore, this is not a valid probability function.

Q1.4
1 Point

$\forall \omega \in \Omega, \mathbb{P}[\omega] = 1/|\Omega|$

- ◉ True
- ○ False

Explanation
The probabilities are non-negative and sum to 1.

Q1.5
1 Point

$\forall \omega \in \Omega, \mathbb{P}[\omega] = (1/\omega)$

- ○ True
- ◉ False

Explanation
The probabilities are 1, $\frac{1}{2}$, and $\frac{1}{3}$, which do not sum to 1.

Q2 Events

2 Points

For this problem, the probability space is $\Omega = \{1, 2, 3, 4\}$ and $\forall \omega \in \Omega, \mathbb{P}[\omega] = \frac{1}{\omega} \frac{12}{25}$.

Q2.1

1 Point

Let A be the event $\{2, 3\}$.

What is $\mathbb{P}[A]$?

- 1
- (selected) $\frac{2}{5}$
- $\frac{3}{5}$
- $\frac{12}{25}$

Explanation

We can sum there respective probabilties: $\frac{12}{25}(\frac{1}{2}) + \frac{12}{25}(\frac{1}{3}) = \frac{10}{25} = \frac{2}{5}$.

Q2.2

1 Point

What is $\mathbb{P}[\overline{A}]$?

- 1
- $\frac{2}{5}$
- (selected) $\frac{3}{5}$
- $\frac{12}{25}$

Explanation

By the defintion of the complement: $\mathbb{P}[\overline{A}] = 1 - \mathbb{P}[A]$.

Q3 Uniform Probability Spaces

2 Points

Suppose you have a fair coin and toss it three times, recording the sequence of tosses for each outcome.

Q3.1

1 Point

What is the cardinality of the sample space Ω?

- 6
- 3
- 8
- 2

Explanation

First rule of counting, an outcome is described by three choices each with 2 possibilities which yields $2^3 = 8$.

Q3.2
1 Point

What is the probability that you have at least two heads?

- $\frac{1}{4}$
- (selected) $\frac{1}{2}$
- $\frac{2}{3}$
- $\frac{3}{8}$

Explanation

One can do this by noting that it is $1 - \mathbb{P}[\text{no heads}] - \mathbb{P}[\text{one heads}]$. There is one outcome with all tails, and three outcomes with one head. Since we know probabilities are uniform, the result is $\frac{4}{8} = \frac{1}{2}$.