Q1 Types of graphs

3 Points

Q1.1 Edge sum

1 Point

What is the sum of all vertex degrees in an n vertex, m edge undirected graph?

- ○ $(n-1)$
- ○ n
- ○ $n(n-1)/2$
- ○ $n\log_2 n$
- ○ $2(n\log_2 n)$
- ○ $(n\log_2 n)/2$
- ○ m
- ○ $2n$
- ◉ $2m$

Explanation

By definition we know each each edge is incident on two vertices. Thus, any one valid edge must increase the sum of vertex degrees by 2, regardless of the vertices it connects. So, the sum of vertex degrees must be $2m$

Q1.2 Tree
1 Point

A tree on n vertices has exactly how many edges?

- (selected) $(n-1)$
- n
- $n(n-1)/2$
- $n\log_2 n$
- $2(n\log_2 n)$
- $(n\log_2 n)/2$

Explanation

A tree is connected and has no cycles, it needs $n-1$ edges to be connected, and adding one would create a cycle.

Q1.3 HyperHyperHyper
1 Point

A hypercube on n vertices has exactly how many edges? (Hint: Is $\log_2 n$ a natural number?)

- $(n-1)$
- n
- $n(n-1)/2$
- $n\log_2 n$
- $2(n\log_2 n)$
- (selected) $(n\log_2 n)/2$

Explanation

A hypercube of dimension d has $n=2^d$ vertices and each vertex has degree $d=\log_2 n$. Thus, the number of edges is the sum of degrees divided by 2 or $2^d d/2 = n\log_2 n/2$.

Q2 Planar graphs
2 Points

Q2.1 Max edge
1 Point

What is the maximum number of edges in an n-vertex planar graph?

- ○ $(n-1)$
- ○ 1
- ○ 0
- ◉ $3n-6$
- ○ 2

Explanation

This is derived in the notes from Euler's formula.

Q2.2 Tree is flat
1 Point

What is the number of faces in a planar drawing of an n-vertex tree?

- ○ $(n-1)$
- ◉ 1
- ○ 0
- ○ $3n-6$
- ○ 2

Explanation

There are no cycles, so the plane is one connected piece. Alternatively, by Euler's formula $f = e - v + 2$, and by the definition of a tree of $e - v = -1$. Thus, $f = 1$

Q3 Graph Coloring

4 Points

A graph G is k-colorable means that the vertices of G can be colored with k colors so that no two adjacent vertices have the same color.

Q3.1

1 Point

K_5 is 4-colorable.

- ○ True
- ● False

Explanation

K_5 requires 5 colors; all vertices are connected to one another, so each must be a different color.

Q3.2

1 Point

$K_{3,3}$ is 4-colorable.

- ● True
- ○ False

Explanation

$K_{3,3}$ is bipartite, so it is 2-colorable and therefore 4-colorable.

Q3.3
1 Point

There exists a graph G with 9 edges and 5 vertices that is *not* 4-colorable.

○ True

◉ False

Explanation

G must be planar; it cannot contain K_5, as it does not have enough edges, and it cannot contain $K_{3,3}$, as it does not have enough vertices. Since G is planar, the 4-color theorem says that G must be 4-colorable.

Q3.4
1 Point

There exists a graph G with 10 edges and 6 vertices that is *not* 4-colorable.

◉ True

○ False

Explanation

Consider K_5 with an isolated vertex; it has 10 edges and 6 vertices, but requires 5 colors.