

Q1

3 Points

For each of the following statements, determine whether each are propositions.

Q1.1

1 Point

2 is prime.

- ☒ Proposition
- ☐ Not a proposition

Explanation

This is a proposition.

Q1.2

1 Point

n is even.

- ☐ Proposition
- ☒ Not a proposition

Explanation

This is not a proposition, since n is a free variable, and is unquantified.

Q1.3

1 Point

$\forall n \in \mathbb{N}, n$ is even.

- ☒ Proposition
- ☐ Not a proposition

Explanation

This is a proposition, since n is quantified.

Q2 Connectives

4 Points

Which of the following are always true, when P and Q are propositions?

Q2.1

1 Point

$$P \vee \neg P$$

- ☒ Always true
- ☐ Not always true

Explanation

P must either be true or false, so this expression is always true.

Q2.2

1 Point

$$(P \vee Q) \implies P$$

- ☐ Always true
- ☒ Not always true

Explanation

If P is false while Q is true, this expression is false.

Q2.3**1 Point**

$$P \equiv (P \vee Q) \wedge (P \vee \neg Q)$$

- ☒ Always true
- ☐ Not always true

Explanation

If Q is true, the RHS simplifies to $\text{True} \wedge P \equiv P$, and if Q is false, the RHS simplifies to $P \wedge \text{True} \equiv P$. Equivalently, we can rewrite the RHS as $P \vee (Q \wedge \neg Q) \equiv P$, since $Q \wedge \neg Q$ is always false.

Q2.4**1 Point**

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

- ☒ Always true
- ☐ Not always true

Explanation

This is true by DeMorgan's laws.

Q3 Quantifiers.

4 Points

Which of the following are always true for predicates $P(n)$ and $Q(n)$ and non-empty universe U ?

Q3.1

1 Point

$$(\forall x \in U)(\neg P(x)) \implies (\exists x \in U)(\neg P(x))$$

- ☒ Always true
- ☐ Not always true

Explanation

This is always true because if the predicate $P(x)$ is false for all x , then there's some x for which it's false.

Q3.2

1 Point

$$\neg(\forall x \in U)(P(x)) \equiv (\exists x \in U)(\neg P(x))$$

- ☒ Always true
- ☐ Not always true

Explanation

This is true because saying "it is not the case that for all x , $P(x)$ is true" is equivalent to claiming "there is some x where $P(x)$ is false".

Q3.3**1 Point**

$$(\forall x \in U)(P(x) \wedge Q(x)) \equiv \left((\forall x \in U)(P(x)) \wedge (\forall x \in U)(Q(x)) \right)$$

- ☒ Always true
- ☐ Not always true

Explanation

This is true because in order for $P(x)$ and $Q(x)$ to be true then both individual predicates must be true.

Q3.4**1 Point**

$$(\forall x \in U)(P(x) \vee Q(x)) \equiv \left((\forall x \in U)(P(x)) \vee (\forall x \in U)(Q(x)) \right)$$

- ☐ Always true
- ☒ Not always true

Explanation

This is false because for $P(x)$ or $Q(x)$ to be true, only one of the two need to be true. One clear counter example is to let U be the set of positive integers, $P(x)$ be the claim " x is odd" and $Q(x)$ be the claim " x is even". We can see that the left hand side is true since every positive integer is either even or odd but the right hand side of the equivalence is false.